Pearson Edexcel

Mark Scheme (Results)

January 2022

Pearson Edexcel International Advanced Level
in Pure Mathematics P4 (WMA14)
Paper 01

Question	Scheme	Marks
1	E.g $xy^2 \rightarrow (...+)2xy\frac{dy}{dx}$ or $x^2y \rightarrow (...+)x^2\frac{dy}{dx}$ but see notes.	M1
	$xy^2 \rightarrow y^2 + kxy\frac{dy}{dx}$ or $x^2y \rightarrow kxy + x^2\frac{dy}{dx}$	dM1
	$y^2 + 2xy\frac{dy}{dx} = 2xy + x^2\frac{dy}{dx}$ oe	A1
	$(x^2 - 2xy)\frac{dy}{dx} = y^2 - 2xy \Rightarrow \frac{dy}{dx} = \frac{y^2-2xy}{x^2-2xy} \Rightarrow \left.\frac{dy}{dx}\right\|_P = \frac{3^2-12}{2^2-12} = ...$ or $3^2 + 2\times2\times3\left.\frac{dy}{dx}\right\|_P = 2\times2\times3 + 2^2\left.\frac{dy}{dx}\right\|_P \Rightarrow \left.\frac{dy}{dx}\right\|_P = \frac{9-12}{4-12} = ...$	M1
	Tangent is $y - 3 = "\frac{3}{8}"(x-2)$	dM1
	$\Rightarrow 3x - 8y + 18 = 0$	A1
		(6)
		(6 marks)

Notes:

M1: For an attempt at implicit differentiation in some form. (Allow y' for $\frac{dy}{dx}$)

For the xy^2 look for a term $kxy\frac{dy}{dx}$ but condone $ky\frac{dy}{dx}$

For the x^2y look for a term $kx^2\frac{dy}{dx}$ but condone $kx\frac{dy}{dx}$

dM1: For attempting the product rule on either xy^2 or x^2y.

Award for $xy^2 \rightarrow y^2 + kxy\frac{dy}{dx}$ **or** $x^2y \rightarrow kxy + x^2\frac{dy}{dx}$

A1: For $y^2 + 2xy\frac{dy}{dx} = 2xy + x^2\frac{dy}{dx}$ or equivalent.

Ignore any spurious "$\frac{dy}{dx} =$" in front of their equation.

M1: Attempts to find $\frac{dy}{dx}$ at *P*. Must substitute **both** *x* and *y* values and rearrange (but can be done in either order). Allow this mark as long as there are at least two terms in $\frac{dy}{dx}$ in their differentiation not including any spurious "$\frac{dy}{dx} =$" There must be some correct substitution.

dM1: Uses their **non-zero** $\frac{dy}{dx}$ at (2, 3) to find the equation of the tangent. **Depends on previous M.**

Look for $y - 3 = "m"(x-2)$ or $y = "m"x + c$ where *m* is their attempt at the gradient at *P*, followed by attempt to find *c* using (2, 3) with the 2 and the 3 **positioned correctly.**

A1: $3x - 8y + 18 = 0$ or any non-zero integer multiple of this equation but with all terms on one side.

Question	Scheme	Marks
2(a)	$\left(1+4x^3\right)^{\frac{1}{3}}=1+\frac{1}{3}\left(...x^{...}\right)+...$	M1
	$\left(1+4x^3\right)^{\frac{1}{3}}=...+\frac{\frac{1}{3}\left(-\frac{2}{3}\right)}{2}\left(4x^3\right)^2+...$	M1
	$=1+\frac{4}{3}x^3+...$ or $=...-\frac{16}{9}x^6$	A1
	$\left(1+4x^3\right)^{\frac{1}{3}}=1+\frac{4}{3}x^3-\frac{16}{9}x^6+...$	A1
		(4)
(b)	$1+4x^3=1+4\times\left(\frac{1}{3}\right)^3=\frac{31}{...}$ **or** $1+\frac{4}{3}\left(\frac{1}{3}\right)^3-\frac{16}{9}\left(\frac{1}{3}\right)^6$	M1
	$\sqrt[3]{31}\approx 3\times\left(1+\frac{4}{3}\left(\frac{1}{3}\right)^3-\frac{16}{9}\left(\frac{1}{3}\right)^6\right)$	dM1
	$=\frac{6869}{2187}$	A1
		(3)
		(7 marks)

Notes:

(a)

M1: For $1\pm\frac{1}{3}\left(4x^{...}\right)$. Allow if the 4 and/or the power of x is incorrect/missing for this mark.

M1: For the correct structure for the third term. Look for the correct binomial coefficient combined with the correct power of $4x^3$ e.g. $\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2}\left(4x^3\right)^2$ but condone missing brackets e.g. $\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2}4x^{3^2}$

A1: For $1+\frac{4}{3}x^3+...$ **or** for $...-\frac{16}{9}x^6$.

Allow mixed fractions e.g. $1\frac{1}{3}$ for $\frac{4}{3}$ but do not allow '+–' for '–'.

A1: For $1+\frac{4}{3}x^3-\frac{16}{9}x^6+...$. **Ignore any extra terms.**

(b)

M1: Attempts $1+4\times\left(\frac{1}{3}\right)^3$ and reaches $\frac{31}{k}$ (this will usually be seen as e.g. $\sqrt[3]{\frac{31}{27}}$) **or** substitutes $x=\frac{1}{3}$ into their expansion from part (a).

dM1: Attempts $3\times$ (their expansion from part (a) with $x=\frac{1}{3}$ substituted)

Allow if they have included more terms in their expansion for both M's. You need to be convinced that they have attempted this calculation and not just found the cube root of 31 on their calculator e.g. 3.1414…

A1: Cao. Correct fraction.

Question	Scheme	Marks
3(a)	$x=3+2\sin t \qquad y=\dfrac{6}{7+\cos 2t} \qquad -\dfrac{\pi}{2}\leqslant t\leqslant\dfrac{\pi}{2}$	
	$y=\dfrac{6}{7+1-2\sin^2 t}$	M1
	$\sin t=\dfrac{x-3}{2}\Rightarrow y=\dfrac{6}{8-2\left(\dfrac{x-3}{2}\right)^2}$	M1A1
	$\Rightarrow y=\dfrac{12}{16-(x-3)^2}=\dfrac{12}{(4-x+3)(4+x-3)}=\dfrac{12}{(7-x)(1+x)}$ *	M1A1*
	$\left(t=-\dfrac{\pi}{2}\Rightarrow x=1, t=\dfrac{\pi}{2}\Rightarrow x=5 \text{ so}\right)$ $p=1$ and $q=5$	B1
		(6)
(a) Way 2	$y=\dfrac{6}{7+1-2\sin^2 t}$	M1
	$y=\dfrac{12}{16-4\sin^2 t}=\dfrac{12}{(4+2\sin t)(4-2\sin t)}$	M1A1
	$=\dfrac{12}{(4+x-3)(4-(x-3))}=\dfrac{12}{(7-x)(1+x)}$ *	M1A1*
	$\left(t=-\dfrac{\pi}{2}\Rightarrow x=1, t=\dfrac{\pi}{2}\Rightarrow x=5 \text{ so}\right)$ $p=1$ and $q=5$	B1
(a) Way 3	$y=\dfrac{12}{(7-x)(1+x)}=\dfrac{12}{(4-2\sin t)(4+2\sin t)}$	M1
	$=\dfrac{12}{16-4\sin^2 t}$	M1A1
	$=\dfrac{12}{16-4\left(\dfrac{1-\cos 2t}{2}\right)}$	M1
	$=\dfrac{12}{16-2+2\cos 2t}=\dfrac{12}{14+2\cos 2t}=\dfrac{6}{7+\cos 2t}=y$ *	A1*
	$\left(t=-\dfrac{\pi}{2}\Rightarrow x=1, t=\dfrac{\pi}{2}\Rightarrow x=5 \text{ so}\right)$ $p=1$ and $q=5$	B1

(a) **Way 4**	$y=\dfrac{6}{7+\cos 2t}\Rightarrow \cos 2t=\dfrac{6}{y}-7\Rightarrow 1-2\sin^2 t=\dfrac{6}{y}-7$	**M1**
	$\sin t=\dfrac{x-3}{2}\Rightarrow \dfrac{6}{y}-7=1-2\left(\dfrac{x-3}{2}\right)^2$	**M1A1**
	$\Rightarrow \dfrac{12}{y}=-x^2+6x+7\Rightarrow y=\dfrac{12}{-x^2+6x+7}=\dfrac{12}{(7-x)(1+x)}$ *	**M1A1***
	$\left(t=-\dfrac{\pi}{2}\Rightarrow x=1, t=\dfrac{\pi}{2}\Rightarrow x=5 \text{ so}\right)$ $p = 1$ and $q = 5$	**B1**
(b)	$\dfrac{12}{(7-x)(1+x)}=\dfrac{A}{7-x}+\dfrac{B}{1+x}\Rightarrow 12=A(x+1)+B(7-x)\Rightarrow A=...,B=...$	**M1**
	So $(y=)\dfrac{3}{2(x+1)}-\dfrac{3}{2(x-7)}$ oe	**A1A1**
		(3)
		(9 marks)

Notes:

(a)

M1: Applies $\cos 2t=\pm 1\pm 2\sin^2 t$ to get y in terms of sin t. This may be implied if they obtain cos $2t$ in terms of x with no explicit sight of sin t.

M1: Rearranges equation for x and substitutes into equation for y.

A1: Correct equation in terms of x and y only.

M1: Simplifies and factorises denominator (usual rules). May have the $\dfrac{1}{2}$ left in the denominator, e.g.

$$y=\frac{6}{8-\frac{1}{2}(x-3)^2}=\frac{6}{8-\frac{1}{2}x^2+3x-\frac{9}{2}}=\frac{6}{-\frac{1}{2}x^2+3x+\frac{7}{2}}=\frac{6}{-\left(\frac{1}{2}x+\frac{1}{2}\right)(x-7)}$$

A1*: Correct result reached with no errors seen.

B1: Correct values for p and q or correct inequality.

Can score anywhere in their response but do not allow $1 \leqslant t \leqslant 5$

(a) Way 2

M1: Applies $\cos 2t=\pm 1\pm 2\sin^2 t$ to get y in terms of sin t

M1: Uses the difference of 2 squares in the denominator

A1: Correct equation with the square completed

M1: Substitutes for sin t in terms of x

A1*: Correct result reached with no errors seen.

B1: Correct values for p and q or correct inequality. **Can score anywhere in their response.**

(a) Way 3

M1: Substitutes for x in terms of t into the given expression for y

M1: Expands denominator.

A1: Correct expression.

M1: Applies $\cos 2t=\pm 1\pm 2\sin^2 t$ to get $\sin^2 t$ in terms of cos $2t$

A1*: Correct result reached with no errors seen and minimal conclusion.

B1: Correct values for p and q or correct inequality. **Can score anywhere in their response.**

(a) Way 4 (similar to Way 1)

M1: Rearranges to get cos $2t$ in terms of y and uses $\cos 2t=\pm 1\pm 2\sin^2 t$

M1: Rearranges equation for x and substitutes to obtain and equation in x and y.
A1: Correct equation in terms of x and y only.
M1: Simplifies and factorises denominator (usual rules).
A1*: Correct result reached with no errors seen.
B1: Correct values for p and q or correct inequality. **Can score anywhere in their response.**
(b)
M1: Correct overall method to apply partial fractions to **the given fraction or equivalent correct fraction** and find at least one constant.
If no method is shown, one correct value can imply the method.
Score M0 for $\dfrac{12}{(7-x)(1+x)} = \dfrac{A}{7-x} + \dfrac{B}{1+x} \Rightarrow 12 = A(7-x) + B(x+1) \Rightarrow A = ..., B = ...$
A1: At least one correct fraction in the required form e.g.
$\dfrac{3}{2(1+x)}, -\dfrac{3}{2(x-7)}$ oe e.g. $\dfrac{\frac{3}{2}}{1+x}, -\dfrac{\frac{6}{4}}{x-7}$ but not e.g. $\dfrac{3}{2x+2}, \dfrac{-3}{2x-14}$
unless a correct form is seen beforehand.
Or a correct pair of values e.g. $a = \dfrac{3}{2}, b = 1$ or $c = \dfrac{3}{2}, d = 1$ or $a = -\dfrac{3}{2}, b = -7$ or $c = -\dfrac{3}{2}, d = -7$
A1: Correct expression for y in the given form $\dfrac{3}{2(x+1)} - \dfrac{3}{2(x-7)}$ (there is no need for the "$y =$")
Accept as in scheme or any equivalents in the correct form, e.g. with 1.5 or equivalent as the numerators.
Or all values correct e.g. $\left(a = \dfrac{3}{2}, b = 1 \text{ and } c = -\dfrac{3}{2}, d = -7\right)$ or $\left(a = -\dfrac{3}{2}, b = -7 \text{ and } c = \dfrac{3}{2}, d = 1\right)$

Note that in (b) some candidates may change the form of y first:

$$\frac{12}{(7-x)(1+x)} = \frac{-12}{(x-7)(1+x)}$$

In such cases the marks can be applied as in the scheme provided the expression is correct.

Question	Scheme	Marks
4(a)	Area of one face $=\frac{1}{2}x\times x\times\sin 60° = \frac{1}{2}x^2\frac{\sqrt{3}}{2}$ oe	M1
	So surface area of icosahedron is $20\times\frac{\sqrt{3}}{4}x^2 = 5\sqrt{3}x^2$ *	A1*
		(2)
(b)	$\frac{dA}{dx}=10\sqrt{3}x$ and $\frac{dV}{dx}=\frac{15}{12}\left(3+\sqrt{5}\right)x^2$	M1
	$\frac{dV}{dA}=\frac{dV}{dx}\times\frac{dx}{dA}=\frac{dV}{dx}\div\frac{dA}{dx}=...$ (oe)	M1
	$=\frac{15\left(3+\sqrt{5}\right)x^2}{12\times10\sqrt{3}x}=\frac{\left(3+\sqrt{5}\right)x}{8\sqrt{3}}$ *	A1*
		(3)
(c)	$\frac{dA}{dt}=0.025$	B1
	$\frac{dV}{dt}=\frac{dV}{dA}\times\frac{dA}{dt}=\frac{\left(3+\sqrt{5}\right)x}{8\sqrt{3}}\times0.025=...$ When $x=2$, $\frac{dV}{dt}=\frac{2\left(3+\sqrt{5}\right)}{8\sqrt{3}}\times0.025=...$	M1
	awrt 0.019 (cm^3s^{-1})	A1
		(3)
		(8 marks)

Notes:

(a)

M1: Any correct method to establish a surd form for the area of one face. Must evaluate trigonometric terms.

Alternatives include e.g. $\frac{1}{2}\times x\times\sqrt{x^2-\left(\frac{1}{2}x\right)^2} = \frac{1}{2}x\sqrt{\frac{3}{4}x^2}$

A1*: Correct result achieved after showing side area of one face.

Examples:

Area of one face $=\frac{1}{2}x\times x\times\sin 60° = \frac{1}{4}x^2\sqrt{3} \Rightarrow SA = 20\times\frac{\sqrt{3}}{4}x^2 = 5\sqrt{3}x^2$ scores M1 A1

$SA = 20\times\left(\frac{1}{2}\times x\times x\sin 60\right)=5\sqrt{3}x^2$ scores M1 A0 for lack of working

(b)

M1: Attempts derivatives of both A and V wrt x. E.g. $\left(\frac{dA}{dx}=\right)\alpha x$ and $\left(\frac{dV}{dx}=\right)\beta x^2$.

Note that $\frac{5}{12}\left(3+\sqrt{5}\right)x^3$ is sometimes expanded first as e.g. $\frac{5}{4}x^3+\frac{5\sqrt{5}}{12}x^3$ and allow this mark if either term is differentiated to βx^2

M1: Applies the correct chain rule with their derivatives. (Condone poor notation – see below)

A1*: Applies the chain rule to achieve the given result, with suitable intermediate step seen with $\frac{\mathrm{d}V}{\mathrm{d}A}=...$ appearing somewhere in their solution.

Condone poor notation e.g. allow $\frac{\mathrm{d}A}{\mathrm{d}t}=10\sqrt{3}x$ and $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{15}{12}\left(3+\sqrt{5}\right)x^2$, $\frac{\mathrm{d}V}{\mathrm{d}A}=\frac{\mathrm{d}V}{\mathrm{d}t}\times\frac{\mathrm{d}t}{\mathrm{d}A}$ etc. for full marks.

Alt:

M1: Eliminates x from the two equations $V=\frac{5}{12}\left(3+\sqrt{5}\right)\left(\frac{A}{5\sqrt{3}}\right)^{\frac{3}{2}}$

M1: Attempts to differentiate $\frac{\mathrm{d}V}{\mathrm{d}A}=\frac{5}{12}\left(3+\sqrt{5}\right)\times\frac{3}{2}\left(\frac{A}{5\sqrt{3}}\right)^{\frac{1}{2}}\times\frac{1}{5\sqrt{3}}$ – look for correct treatment of the $5\sqrt{3}$ as well as correct power.

A1: Achieves correct result with no errors.

(c)

B1: Interprets the rate of change correctly to state or clearly imply by working that $\frac{\mathrm{d}A}{\mathrm{d}t}=0.025$

M1: Applies the chain rule appropriately **to find a value for** $\frac{\mathrm{d}V}{\mathrm{d}t}$ using $x = 2$

A1: awrt 0.019 condone lack of units.

Question	Scheme	Marks
5(a)	$\frac{dx}{dt}=\frac{1}{2}(9-4t)^{-\frac{1}{2}}\times -4=\left(\frac{-2}{\sqrt{9-4t}}\right)$	M1
	$A=\int_{(x=0)}^{(x=3)} y\frac{dx}{dt}(dt)=\int_{\left(t=\frac{9}{4}\right)}^{(t=0)}\frac{t^3}{\sqrt{9+4t}}\times\text{"}\frac{-2}{\sqrt{9-4t}}\text{"}(dt)$	M1
	$=-2\int_{\left(\frac{9}{4}\right)}^{(0)}\frac{t^3}{\sqrt{81-16t^2}}(dt)$ or e.g. $-2\int_{\left(\frac{9}{4}\right)}^{(0)}\frac{t^3}{\sqrt{(9-4t)(9+4t)}}(dt)$ **Depends on both previous M marks.**	ddM1
	$=2\int_0^{\frac{9}{4}}\frac{t^3}{\sqrt{81-16t^2}}dt$	A1
		(4)
(b)	$\frac{du}{dt}=-32t$ or $du=-32t\,dt$ oe or $u=81-16t^2\Rightarrow t^2=\frac{1}{16}(81-u)\Rightarrow t=\frac{1}{4}(81-u)^{\frac{1}{2}}\Rightarrow\frac{dt}{du}=-\frac{1}{8}(81-u)^{-\frac{1}{2}}$	B1
	$A=2\int_0^{\frac{9}{4}}\frac{t^2}{\sqrt{81-16t^2}}t\,dt=2\int_{(81)}^{(0)}\frac{(81-u)}{16\sqrt{u}}\frac{du}{-32}$ or $A=2\int_0^{\frac{9}{4}}\frac{t^2}{\sqrt{81-16t^2}}t\,dt=2\int_{(81)}^{(0)}\frac{\left(\frac{1}{4}(81-u)^{\frac{1}{2}}\right)^3}{\sqrt{u}}\times-\frac{1}{8}(81-u)^{-\frac{1}{2}}du$	M1
	$=k\int_{(81)}^{(0)}81u^{-\frac{1}{2}}-u^{\frac{1}{2}}du$	A1ft
	$=K\left[\frac{81u^{\frac{1}{2}}}{\frac{1}{2}}-\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]_{(81)}^{(0)}$	M1
	$=-\frac{1}{256}\left(0-\left(162\sqrt{81}-\frac{2}{3}\sqrt{81}^3\right)\right)=...$ **Depends on all previous M marks.**	ddM1
	$=\frac{243}{64}$	A1
		(6)
		(10 marks)

Notes:

(a)

M1: Attempts $\frac{dx}{dt}$ achieving the form $k(9-4t)^{-\frac{1}{2}}$ oe

M1: Applies the correct formula for the area, $A=\int y\frac{dx}{dt}dt$, with y and their $\frac{dx}{dt}$

(no need for limits for this mark and no need for "dt").

ddM1: Combines terms to a single fraction with one square root.

(no need for limits for this mark and no need for "dt").

Depends on both previous M marks.

A1: Applies difference of squares, or expands, and deals with limits correctly to achieve correct result from correct working.

The "dt" must have made at least one appearance before the final line.

(b)

B1: Correct statement of $\frac{du}{dt}$ or $\frac{dt}{du}$ or any correct equation connecting du with dt

M1: Complete substitution made into the integrand given in part (a) with their K or the letter K or a made up K e.g. $K = 1$ to obtain an integral in terms of u only.

Don't be concerned with the limits at this stage.

Condone the omission of the "du".

A1ft: For any multiple of $\int\left(81u^{-\frac{1}{2}}-u^{\frac{1}{2}}\right)du$. Limits are not needed.

Condone the omission of the "du".

M1: Integrates an expression of the form $ku^{-\frac{1}{2}}+lu^{\frac{1}{2}}\rightarrow Au^{\frac{1}{2}}+Bu^{\frac{3}{2}}$

ddM1: Applies correct limits to their integral either way round. Correct limits are 0 and 81 if substituting into their integral in terms of u or 0 and $\frac{9}{4}$ if reverting to t.

Depends on all previous M marks.

A1: Correct answer. Correct work leading to $-\frac{243}{64}$ scores A0 but allow "hence area is $\frac{243}{64}$"

Note that the question says "or otherwise" so some alternatives are:

Uses the substitution $u=\sqrt{81-16t^2}$:

B1: Correct derivative statement $\frac{du}{dt}=\frac{1}{2}\left(81-16t^2\right)^{-\frac{1}{2}}\times-32t$ oe such as $2u\,du=-32t\,dt$

M1A1ft: Full substitution leading to $2\int\frac{81-u^2}{16u}\frac{2u}{-32}du=-\frac{1}{128}\int 81-u^2\,du$

ddM1: Integrates to $Ku-Lu^3$ **M1:** Applies correct limits 0 and 81 (or returns to t)

Depends on all previous M marks.

A1: Correct.

Integration by parts:

B1: For $\int\frac{t}{\sqrt{81-16t^2}}dt=-\frac{1}{16}\sqrt{81-16t^2}$ seen or implied in working.

M1A1ft: Correct application of parts in the right direction

$$2\int t^2 \times \frac{t}{\sqrt{81-16t^2}}\mathrm{d}t = 2t^2 \times -\frac{1}{16}\sqrt{81-16t^2} - 2\int 2t \times -\frac{1}{16}\sqrt{81-16t^2}\,\mathrm{d}t$$
M1: Completes the integration process $= -\frac{t^2}{8}\sqrt{81-16t^2} + \frac{1}{4} \times -\frac{\sqrt{81-16t^2}^{3}}{48}$
ddM1: Applies limits of 0 and $\frac{9}{4}$. **Depends on all previous M marks.**
A1: Correct answer.

Question	Scheme	Marks
6	Assume the sequence is geometric	B1
	So $\left(r=\right)\dfrac{1+2k}{k}=\dfrac{3+3k}{1+2k}$	M1
	$\Rightarrow (1+2k)^2=k(3+3k)\Rightarrow k^2+k+1=0$	A1
	But $k^2+k+1=\left(k+\dfrac{1}{2}\right)^2-\dfrac{1}{4}+1\geqslant\dfrac{3}{4}>0$ (since $\left(k+\dfrac{1}{2}\right)^2\geqslant 0$ for all (real) k)	dM1
	This is a contradiction and hence the original assumption is not true. The sequence is not geometric.	A1
		(5)
		(5 marks)

Notes:

(a)

B1: States an appropriate assumption to set up the contradiction.

M1: Uses the assumption to set up an equation in *k* only.

Allow equivalent work e.g. $kr=1+2k,\ kr^2=3+3k\Rightarrow 3+3k=k\left(\dfrac{1+2k}{k}\right)^2$

Allow use of ≠ for = e.g. $\sqrt{\dfrac{3+3k}{k}}\neq\dfrac{1+2k}{k}$

This may be implied by e.g. $\dfrac{1+2k}{k}$ is not the same as $\dfrac{3+3k}{1+2k}$.

A1: Reaches a correct quadratic equation in *k*, need not be all on one side, but terms in *k* and k^2 should be collected. Allow use of ≠ for = e.g. $k^2+k+1\neq 0$

dM1: Completes the square, considers the discriminant or other valid means used to reach a point where a contradiction can be deduced. E.g. as scheme, or $b^2-4ac=1^2-4(1)(1)=-3<0$ may be used. Accept use of calculator to give roots $k=\dfrac{-1\pm i\sqrt{3}}{2}$ so *k* is not real, which contradicts *k* being a member of the real sequence.

Depends on the previous M.

A1: Correct work leading to a contradiction with deduction of a contradiction made and conclusion given. **This mark is available even if B0 is given at the start. So 01111 is possible.**

If they are using the discriminant or calculator route then there is no need to mention "real" as long as they conclude that e.g. the geometric sequence is not possible. This can score both the dM1 and A1.

Question	Scheme	Marks
7(a)	$V=(\pi)\int_{(0)}^{(4)}\left(\frac{1}{4}(4-x)e^{x}\right)^{2}\,dx$	M1
	$\int(f(x))^{2}\,dx=\frac{1}{16}\int(4-x)^{2}e^{2x}dx=\ldots$	M1
	$V=\frac{\pi}{16}\int_{0}^{4}\left(x^{2}-8x+16\right)e^{2x}\,dx$	A1
		(3)
(b)	$=\left("\frac{\pi}{16}"\right)\left((x^{2}-8x+16)\frac{e^{2x}}{2}-\int(2x-8)\times\frac{e^{2x}}{2}dx\right)$	M1 A1
	$=\left("\frac{\pi}{16}"\right)\left(\left(x^{2}-8x+16\right)\frac{e^{2x}}{2}-\left[(2x-8)\frac{e^{2x}}{4}-\int 2\times\frac{e^{2x}}{4}dx\right]\right)$ $=\left("\frac{\pi}{16}"\right)\left(\left(x^{2}-8x+16\right)\frac{e^{2x}}{2}-(2x-8)\frac{e^{2x}}{4}+\frac{e^{2x}}{4}\right)$ **Depends on the first method mark.**	dM1
	$V="\frac{\pi}{16}"\left[\left(x^{2}-8x+16\right)\frac{e^{2x}}{2}-(2x-8)\frac{e^{2x}}{4}+\frac{e^{2x}}{4}\right]_{0}^{4}="\frac{\pi}{16}"\left(0+0+\frac{e^{8}}{4}-\frac{16}{2}-\frac{8}{4}-\frac{1}{4}\right)$ **Depends on both previous method marks.**	ddM1
	$=\frac{\pi}{64}\left(e^{8}-41\right)$ (cm^{3})	A1
		(5)
(b) Way 2	$=\left("\frac{\pi}{16}"\right)\left((4-x)^{2}\frac{e^{2x}}{2}+\int(4-x)e^{2x}\,dx\right)$	M1 A1
	$=\left("\frac{\pi}{16}"\right)\left((4-x)^{2}\frac{e^{2x}}{2}+\frac{1}{2}(4-x)e^{2x}+\int\frac{1}{2}e^{2x}\,dx\right)$ $=\left("\frac{\pi}{16}"\right)\left((4-x)^{2}\frac{e^{2x}}{2}+\frac{1}{2}(4-x)e^{2x}+\frac{1}{4}e^{2x}\right)$ **Depends on the first method mark.**	dM1
	$=\left("\frac{\pi}{16}"\right)\left[(4-x)^{2}\frac{e^{2x}}{2}+\frac{1}{2}(4-x)e^{2x}+\frac{1}{4}e^{2x}\right]_{0}^{4}=\left("\frac{\pi}{16}"\right)\left(\frac{1}{4}e^{8}-8-2-\frac{1}{4}\right)$ **Depends on both previous method marks.**	ddM1
	$=\frac{\pi}{64}\left(e^{8}-41\right)$ (cm^{3})	A1

(b) Way 3		
	$\int_0^4 (x^2-8x+16)e^{2x}\,dx = \int_0^4 x^2e^{2x}\,dx - \int_0^4 8xe^{2x}\,dx + \int_0^4 16e^{2x}\,dx$	
	$=\left(\frac{"\pi"}{16}\right)\left(x^2\frac{e^{2x}}{2} - \int 2x\times\frac{e^{2x}}{2}dx + ...\right)$	M1A1
	$\int_0^4 x^2e^{2x}\,dx - \int_0^4 8xe^{2x}\,dx + \int_0^4 16e^{2x}\,dx$ $=\left(\frac{"\pi"}{16}\right)\left(x^2\frac{e^{2x}}{2} - \left\{x\times\frac{e^{2x}}{2} - \int\frac{e^{2x}}{2}dx\right\} - 8\left\{x\times\frac{e^{2x}}{2} - \int\frac{e^{2x}}{2}dx\right\} + 8e^{2x}\right)$ **Depends on the first method mark.**	dM1
	$\left(\frac{\pi}{16}\right)\left[\frac{x^2e^{2x}}{2} - \frac{9xe^{2x}}{2} + \frac{41e^{2x}}{4}\right]_0^4 = \frac{\pi}{16}\left(8e^8 - 18e^8 + \frac{41}{4}e^8 - \frac{41}{4}\right)$ **Depends on both previous method marks.**	ddM1
	$=\frac{\pi}{64}(e^8 - 41)$ (cm^3)	A1
		(8 marks)

Notes:

(a)

M1: Sets up the volume using the volume formula, but allow limits and π and the "dx" to be missing. No need to square f(x) for this mark as long as the intent to do so is clear. There should be an integral

M1: Attempts to square f(x) (need not be part of an integral). The power of e should be correct and the bracket expanded, but allow if the $\frac{1}{4}$ is not squared.

A1: Correct expression for the integral, π and limits must be included, and no errors seen in working so the "dx" must be present throughout.

(b)

M1: Attempts integration by parts the correct way at least once on the given integral to obtain

$$\alpha(x^2-8x+16)e^{2x} \pm \beta\int(ax-b)e^{2x}\,dx$$

A1: Correct first application of parts: $(x^2-8x+16)\frac{e^{2x}}{2} - \int(2x-8)\times\frac{e^{2x}}{2}dx$ or equivalent.

dM1: Attempts integration by parts again in the same direction on $\int(2x-8)\times\frac{e^{2x}}{2}dx$ and proceeds to complete the integration. Look for $K\left(...+\alpha(ax-b)e^{2x} \pm \beta\int e^{2x}dx\right)$.

ddM1: Applies the limits to their integral and subtracts, (but this need not have the K or π for this mark.) May be done in stages, so look for reaching a value using correct limits on each part of their integral.

A1: Correct answer achieved from correct work and the π must have been introduced before the final step, it should have been part of their answer to (a). Condone lack of units.

Way 2

M1: Attempts integration by parts the correct way at least once on the given integral to obtain

$$\alpha(4-x)^2 e^{2x} \pm \beta\int (4-x)e^{2x}\,dx.$$

A1: Correct first application of parts: $(4-x)^2\frac{e^{2x}}{2}+\int (4-x)e^{2x}\,dx$ or equivalent.

dM1: Attempts integration by parts again in the same direction on $\int (4-x)e^{2x}\,dx$ **and proceeds to complete the integration**. Look for $K\left(...+\alpha(4-x)e^{2x} \pm \beta\int e^{2x}dx\right)$ for the parts.

ddM1: Applies the limits to their integral and subtracts, (but this need not have the K or π for this mark.) May be done in stages, so look for reaching a value using correct limits on each part of their integral.

A1: Correct answer achieved from correct work and the π must have been introduced before the final step, it should have been part of their answer to (a). Condone lack of units.

Way 3

M1: Splits the integral and attempts integration by parts the correct way at least once on $\int x^2e^{2x}\,dx$.

Look for $\alpha x^2e^{2x} \pm \beta\int xe^{2x}\,dx$.

A1: Correct first application of parts: $x^2\frac{e^{2x}}{2}-\int 2x\frac{e^{2x}}{2}dx$ or equivalent.

dM1: Applies parts a second time on both occurrences of $\int x\,e^{2x}\,dx$ and completes the integration.

ddM1: Applies the limits to their integral and subtracts, (but this need not have the K or π for this mark.) May be done in stages, so look for reaching a value using correct limits on each part of their integral.

A1: Correct answer achieved from correct work and the π must have been introduced before the final step, it should have been part of their answer to (a). Condone lack of units.

Question	Scheme	Marks
8(a)	Attempts $\pm\left(\begin{pmatrix}6\\0\\7\end{pmatrix}-\begin{pmatrix}6\\6\\2\end{pmatrix}\right)=\pm\begin{pmatrix}0\\-6\\5\end{pmatrix}$	M1
	$\mathbf{r}=\begin{pmatrix}6\\6\\2\end{pmatrix}+\lambda\begin{pmatrix}0\\-6\\5\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}6\\0\\7\end{pmatrix}+\lambda\begin{pmatrix}0\\6\\-5\end{pmatrix}$ oe	A1
		(2)
(b)	Meet if $\begin{pmatrix}6\\6\\2\end{pmatrix}+\lambda\begin{pmatrix}0\\-6\\5\end{pmatrix}=\begin{pmatrix}3\\1\\4\end{pmatrix}+\mu\begin{pmatrix}1\\5\\9\end{pmatrix}$ so $\begin{cases}6=3+\mu\\6-6\lambda=1+5\mu\\2+5\lambda=4+9\mu\end{cases}$	M1
	From first equation $\mu=3$ Alt: solves equations 2 and 3 to give either $\mu=\frac{13}{79}$ or $\lambda=\frac{55}{79}$	A1
	Then need $\lambda=\frac{5-5\times3}{6}=-\frac{5}{3}$ from 2^{nd} equation, $\lambda=\frac{2+9\times3}{5}=\frac{29}{5}$ from 3^{rd} equation Alt: checks their μ in the first equation	M1
	Values of λ do not agree and hence lines do not meet.	A1
		(4)
(c)	$\overrightarrow{OC}=\begin{pmatrix}3\\1\\4\end{pmatrix}-\begin{pmatrix}1\\5\\9\end{pmatrix}\left(=\begin{pmatrix}2\\-4\\-5\end{pmatrix}\right)$	M1
	$\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=\begin{pmatrix}2\\-4\\-5\end{pmatrix}-\begin{pmatrix}6\\6\\2\end{pmatrix}=\begin{pmatrix}-4\\-10\\-7\end{pmatrix}$ Accept ±. **Depends on first mark.**	dM1
	$\frac{\pm\overrightarrow{AC}\bullet(\mathbf{i}+5\mathbf{j}+9\mathbf{k})}{\lvert\overrightarrow{AC}\rvert\lvert\mathbf{i}+5\mathbf{j}+9\mathbf{k}\rvert}=\pm\frac{-4\times1+-10\times5+-7\times9}{\sqrt{16+100+49}\sqrt{1+25+81}}$ **Depends on both previous marks.**	ddM1A1
	$\Rightarrow\theta=\arccos\left\lvert\frac{-117}{\sqrt{165}\sqrt{107}}\right\rvert=28.3°$ or e.g. $\Rightarrow\theta=90°-\arcsin\left\lvert\frac{-117}{\sqrt{165}\sqrt{107}}\right\rvert=28.3°$	A1
		(5)
		(11 marks)

Notes:
(a) **M1:** Attempts the difference between the given two vectors. Implied by 2 out of three correct coordinates if no method is shown. Can be either way round. **A1:** Any correct equation for the line in the form given and **must have** "**r** = ….". Two examples are shown in the scheme, but they may use any point on the line as the starting point, and any multiple of the direction vector. Allow in **i**, **j**, **k** form but not e.g. $\mathbf{r}=\begin{pmatrix}6\mathbf{i}\\6\mathbf{j}\\2\mathbf{k}\end{pmatrix}+\lambda\begin{pmatrix}0\\-6\mathbf{j}\\5\mathbf{k}\end{pmatrix}$ but condone missing brackets on the position vector so condone e.g. $\mathbf{r}=\begin{matrix}6\\6\\2\end{matrix}+\lambda\begin{pmatrix}0\\-6\\5\end{pmatrix}$ but not on the direction so $\mathbf{r}=\begin{pmatrix}6\\6\\2\end{pmatrix}+\lambda\begin{matrix}0\\-6\\5\end{matrix}$ is A0 **(b)** **M1:** Equates their line and the given line and extracts at least one equation in λ **and** μ **A1:** **"**Correct" value for one parameter. This will depend on which equation(s) they solve – see main scheme **but must follow M1**. **M1:** Substitutes into both other equations to compare results. **A1:** Correct values for λ found with conclusion that the lines do not meet. **Alt:** If equation 1 is not first used to find μ then the 1st A mark can be awarded for any correct value for λ or μ from the equations they solve, and the M mark for checking their solutions for consistency in the remaining equation. **For the final A, all work must have been correct with a suitable conclusion made.** **When checking, allow e.g.** $2-\frac{25}{3}\neq 4+27$ **so they do not meet. I.e. the calculations may not be fully carried out as long as the result is obvious. However if either side is subsequently evaluated incorrectly, withhold the final mark.** FYI: equations (1) and (2) give $\lambda=-\frac{5}{3}$ or $\mu=3$; equations (1) and (3) give $\lambda=\frac{29}{5}$ or $\mu=3$ and equations (2) and (3) give $\lambda=\frac{55}{79}$ or $\mu=\frac{13}{79}$ (use of the negative direction vectors reverse the signs of λs) If $\mathbf{r}=\begin{pmatrix}6\\0\\7\end{pmatrix}+\lambda\begin{pmatrix}0\\6\\-5\end{pmatrix}$ is used equations (1) and (2) give $\lambda=\frac{8}{3}$ or $\mu=3$; equations (1) and (3) give $\lambda=-\frac{24}{5}$ or $\mu=3$ and equations (2) and (3) give $\lambda=\frac{24}{79}$ or $\mu=\frac{13}{79}$ (use of the negative direction vectors reverse the signs of λs) **Note that the maximum marks in (b) if their line in (a) is incorrect is 1010**

There will be many different approaches to part (b).
The general structure for marking is as follows:
M1: Equates lines and obtains at least one equation in λ and μ
A1: Correct value for one of the parameters
M1: Attempts the checking process to show the lines do not meet
A1: All correct with a conclusion

(c)

M1: Substitutes $\mu = -1$ into l_2 to find the coordinates for C or the vector $\overrightarrow{OC}$

dM1: Uses their $\overrightarrow{OC}$ to find $\pm\overrightarrow{AC}$. Look for an attempt at the difference of vectors with at least two correct coordinates.

ddM1: Attempts $\dfrac{\pm\overrightarrow{AC}\bullet(\mathbf{i}+5\mathbf{j}+9\mathbf{k})}{\left|\overrightarrow{AC}\right|\left|\mathbf{i}+5\mathbf{j}+9\mathbf{k}\right|}$ with their $\overrightarrow{AC}$

This requires an attempt at the scalar product of their $\overrightarrow{AC}$ with $\mathbf{i}+5\mathbf{j}+9\mathbf{k}$ in the numerator with at least 2 "components" correct if no method is shown, and correct attempts at the product of the magnitudes of $\overrightarrow{AC}$ and the direction of $\mathbf{i}+5\mathbf{j}+9\mathbf{k}$ in the denominator.

A1: A correct expression simplified or unsimplified e.g. $\pm\left(\dfrac{-4\times1+-10\times5+-7\times9}{\sqrt{16+100+49}\sqrt{1+25+81}}\right)$, $\pm\dfrac{117}{\sqrt{17655}}$

A1: Correct answer. Accept awrt 28.3

Question	Scheme	Marks
9(a)	$\frac{\mathrm{d}}{\mathrm{d}y}(1+2\ln y)^{-2}=\alpha(1+2\ln y)^{-3}\times\frac{\beta}{y}=\frac{K}{y(1+2\ln y)^3}$	M1
	$=\frac{-4}{y(1+2\ln y)^3}$ oe	A1
		(2)
(b)	$3\operatorname{cosec}(2x)\frac{\mathrm{d}y}{\mathrm{d}x}=y(1+2\ln y)^3$ $\Rightarrow\int ...\mathrm{d}y=\int k\sin(2x)\,\mathrm{d}x$ **or** $\int\frac{k}{y(1+2\ln y)^3}\mathrm{d}y=\int ...\mathrm{d}x$ **and** $\Rightarrow\int ...\mathrm{d}y=\int k\sin(2x)\,\mathrm{d}x\Rightarrow ...=A\cos 2x$ oe **or** $\Rightarrow\int\frac{k}{y(1+2\ln y)^3}\mathrm{d}y=\int ...\mathrm{d}x\Rightarrow\frac{A}{(1+2\ln y)^2}=...$	M1
	$\Rightarrow\int\frac{A}{y(1+2\ln y)^3}\mathrm{d}y=\int B\sin(2x)\,\mathrm{d}x\Rightarrow\frac{C}{(1+2\ln y)^2}=D\cos 2x$ oe	M1
	One side integrated **<u>correctly</u>** $\int\frac{3}{y(1+2\ln y)^3}\mathrm{d}y=-\frac{3}{4(1+2\ln y)^2},\int\frac{1}{y(1+2\ln y)^3}\mathrm{d}y=-\frac{1}{4(1+2\ln y)^2}$ or $\int\frac{1}{3}\sin(2x)\,\mathrm{d}x=-\frac{1}{6}\cos(2x)$ oe, $\int\sin(2x)\,\mathrm{d}x=-\frac{1}{2}\cos(2x)$ oe	A1
	$-\frac{3}{4(1+2\ln y)^2}=-\frac{1}{2}\cos(2x)(+c)$ or $-\frac{1}{4(1+2\ln y)^2}=-\frac{1}{6}\cos(2x)(+c)$	A1
		(4)
(c)	$y=1,x=\frac{\pi}{6}\Rightarrow-\frac{3}{4(1+2\ln 1)}=-\frac{1}{2}\cos\left(\frac{\pi}{3}\right)+c\Rightarrow c=...$	M1
	$-\frac{3}{4(1+2\ln y)^2}=-\frac{1}{2}\cos(2x)-\frac{1}{2}$ or $-\frac{1}{4(1+2\ln y)^2}=-\frac{1}{6}\cos(2x)-\frac{1}{6}$	A1
	$-\frac{3}{4(1+2\ln y)^2}=-\frac{1}{2}\cos(2x)-\frac{1}{2}\Rightarrow\frac{3}{4(1+2\ln y)^2}=\frac{1}{2}+\frac{1}{2}\cos(2x)=\cos^2 x$	M1
	$\Rightarrow(1+2\ln y)^2=\frac{3}{4}\sec^2 x\Rightarrow\ln y=\frac{1}{2}\left(\frac{\sqrt{3}}{2}\sec x-1\right)$ **Depends on both previous method marks.**	ddM1
	$\Rightarrow y=\mathrm{e}^{\frac{\sqrt{3}}{4}\sec x-\frac{1}{2}}$	A1
		(5)

(11 marks)

Notes:

(a)

M1: Differentiates to achieve the form $\dfrac{K}{y(1+2\ln y)^3}$.

You can ignore what they call it so just look for the expression.

A1: Correct derivative, accept any equivalent form, need not be simplified.

You can ignore what they call it so just look for the expression.

(b)

M1: Attempts to separate the variables with one side correct ignoring coefficients and with at least one side integrated to the correct form. The separation may have $\dfrac{k}{\operatorname{cosec} 2x}$ rather than $k\sin 2x$.

Do not be concerned about the presence or absence of the "dx" and "dy" or even if they are wrongly position – just look for the expressions appearing on either side.

M1: Attempt to separate the variables with both sides correct ignoring coefficients and with both sides integrated to the correct form. The separation may have $\dfrac{k}{\operatorname{cosec} 2x}$ rather than $k\sin 2x$.

Do not be concerned about the presence or absence of the "dx" and "dy" or even if they are wrongly position – just look for the expressions appearing on either side.

A1: One side integrated **correctly**.

For the LHS this will be either $-\dfrac{1}{4(1+2\ln y)^2}$ or $-\dfrac{3}{4(1+2\ln y)^2}$

For the RHS this will be either $-\dfrac{1}{2}\cos(2x)$ or $-\dfrac{1}{6}\cos(2x)$ but note that if sin 2x is changed to 2sin x cos x first, the integration may appear as e.g. $\dfrac{1}{3}\sin^2 x$ or $-\dfrac{1}{3}\cos^2 x$ or $\sin^2 x$ or $-\cos^2 x$

A1: A **fully correct** general solution, need not be simplified and a constant of integration is not required.

Note that M1M0A1A0 is possible as a mark profile.

(c)

<u>Note that if a candidate rearranges to an explicit form in part (b) the work can be credited in part (c) but there is no credit for work for part (c) in part (b) unless part (c) is attempted.</u>

M1: Substitutes the boundary values into their equation to find their constant which does not to be evaluated for this mark.

This may be done later in the solution, but they must have a constant in the equation.

A1: Correct equation in any form with their constant evaluated. **Must follow A1 in (b).**

M1: Applies cos 2x = $\pm 2\cos^2 x \pm 1$ at some point in their solution.

May see attempts to rearrange to $y = \ldots$ first **<u>and note that if candidates integrate the sin 2x in part (b) to …cos²x then they can score this mark by implication if part (c) is attempted.</u>**

ddM1: Rearranges the equation, **including square rooting**, to reach ln$y$ = …

Depends on both previous method marks.

A1: Correct answer achieved.

<u>You may see candidates rearrange first in (b) to achieve e.g.</u>

$$C-\frac{3}{4(1+2\ln y)^2}=-\frac{1}{2}\cos(2x)\Rightarrow C+\frac{1}{2}\cos(2x)=\frac{3}{4(1+2\ln y)^2}\Rightarrow (1+2\ln y)^2=\frac{3}{4C+2\cos(2x)}$$

$$\Rightarrow \ln y=\frac{1}{2}\sqrt{\frac{3}{4C+2\cos(2x)}}-\frac{1}{2}\Rightarrow y=\exp\left(\frac{1}{2}\sqrt{\frac{3}{4C+2\cos(2x)}}-\frac{1}{2}\right)$$

The first M in (c) will be scored when they substitute the given conditions into their rearranged equation that contains a constant and the first A mark for a correct equation. They will also score the second M if they use the double angle formula. The dM mark will be as described e.g. Rearranges the equation, **including square rooting**, to reach ln$y$ = … and d**epends on both previous method marks.**

There are no marks in (c) for substituting the initial conditions into the given answer to find the value of *A*. If you see any attempts to use this answer to show it satisfies the differential equation in (b) let your TL know.